A-LEVEL

Mathematics

Pure Core 1 – MPC1

Mark scheme

6360

June 2015

Version/Stage: 1.0: Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
(a)	$y = \pm\frac{3}{5}x + \ldots$	M1		$y = -\frac{3}{5}x + \frac{7}{5}$ for guidance
	(Gradient AB =) $-\frac{3}{5}$	A1	2	
(b)	Grad of perp = $\frac{5}{3}$	M1		**FT** negative reciprocal of their **(a)**
	$y + 3 = \frac{5}{3}(x+2)$	A1		**any** correct form with $--$ simplified to $+$ eg $y = \frac{5}{3}x + c,\ c = \frac{1}{3}$
	$5x - 3y + 1 = 0$	A1	3	integer coefficients with all terms on one side of equation & "=0"
(c)	$3x + 5y = 7$ & $2x - 3y = 30$ eg $9x + 10x = 21 + 150$	M1		**correct** equations used and **correct** elimination of x or y eg $19x = 171$ or $19y = -76$
	$x = 9$ **or** $x = \frac{171}{19}$ **or** $y = -4$ **or** $y = \frac{-76}{19}$	A1		either x or y correct in any equivalent form
	$x = 9$ **and** $y = -4$	A1	3	$(9, -4)$ both written as integers
	Total		**8**	

(a) Do not penalise incorrect rearrangement if correct gradient is stated.

Example $y = -\frac{3}{5}x + 7$ so grad = $-\frac{3}{5}$ scores **M1 A1**

NMS (grad AB = **)** $-\frac{3}{5}$ earns 2 marks. **NMS (**grad AB =) $\frac{3}{5}$ earns **M1 A0** .

NMS Award **M1 A0** only for "gradient $= -\frac{3}{5}x$".

May use two **correct** points eg (–1,2) and (–6,5) then $\frac{5-2}{-6--1}$ scores **M1** (must be correct unsimplified) with **A1** for $-\frac{3}{5}$

(b) Condone $0 = 6y - 10x - 2$ etc for final **A1**, but not $3y - 5x = 1$ etc

(c) $2\left(\frac{7}{3} - \frac{5y}{3}\right) - 3y = 30$ earns **M1,** however $2\left(\frac{7}{3} + \frac{5y}{3}\right) - 3y = 30$, for example, scores **M0.**

Accept any equivalent form for first **A1** but must have $x = 9$ **and** $y = -4$ for final **A1**.

Q2	Solution	Mark	Total	Comment
	$\dfrac{4\sqrt{5}-2\sqrt{3}}{\sqrt{5}-\sqrt{3}}$	**B1**		or $\dfrac{2\sqrt{3}-4\sqrt{5}}{\sqrt{3}-\sqrt{5}}$
	$\times\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$	**M1**		multiplying top & bottom by conjugate of **their** denominator
	(Numerator =) $20+4\sqrt{15}-2\sqrt{15}-6$	**A1**		$14+2\sqrt{15}$
	(Denominator =) $\left(5-\sqrt{5}\sqrt{3}+\sqrt{5}\sqrt{3}-3=\right)$ 2	**B1**		must be seen as denominator $\dfrac{14+2\sqrt{15}}{2}$
	(Gradient =) $7+\sqrt{15}$	**A1cso**	5	
	Total		**5**	
	NO MISREADS ALLOWED IN THIS QUESTION Condone multiplication by $\sqrt{5}+\sqrt{3}$ instead of $\times\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ for **M1 only if** subsequent working shows multiplication by both numerator and denominator – otherwise **M0** Must have $\sqrt{15}$ and not just $\sqrt{3}\sqrt{5}$ for first **A1** An error in the denominator such as $5-\sqrt{8}+\sqrt{8}-3=2$ should be given **B0** and it would then automatically lose the final **A1cso** May use alternative conjugate $\times\dfrac{-\sqrt{5}-\sqrt{3}}{-\sqrt{5}-\sqrt{3}}$ **M1 ;** numerator = $-14-2\sqrt{15}$ **A1** etc **M1** is available if gradient expression is incorrect, provided it is a quotient of two surd expressions and the conjugate of their denominator is used. **SC2** for $\dfrac{\sqrt{5}-\sqrt{3}}{4\sqrt{5}-2\sqrt{3}}\times\dfrac{4\sqrt{5}+2\sqrt{3}}{4\sqrt{5}+2\sqrt{3}}=\dfrac{****}{68}$			

Q3	Solution	Mark	Total	Comment
(a)	$\left(\frac{dy}{dx}=\right)\ 4x^3+6x$	M1 A1		one term correct all correct (no +c etc)
	when $x=-1$, $\frac{dy}{dx}=-4-6=-10$	m1		sub $x=-1$ correctly into "their" $\frac{dy}{dx}$ and evaluate correctly
	$y-6=-10(x+1)$	A1cso	4	**any** correct form with – – simplified to + **eg** $y=-10x+c,\quad c=-4$
(b)(i)	$\frac{x^5}{5}+\frac{3x^3}{3}+2x$	M1 A1		two terms correct all correct (may have +c)
	$F(2)-F(-1)$	m1		clear attempt to use correct limits correctly
	$\left[\frac{32}{5}+8+4\right]-\left[-\frac{1}{5}-1-2\right]$	A1		correct unsimplified must evaluate 2^5; $(-1)^3$ etc
	$=21.6$	A1cso	5	$21\frac{3}{5}$ **;** $\frac{108}{5}$ **OE**
(ii)	(Area of trapezium =) 54	B1		allow 18+36 or 90 – 36
	(Shaded area =) 54 – 21.6	M1		Area of **trapezium** – |*their* value from **(b)(i)**|
	$=32.4$	A1cso	3	$32\frac{2}{5}$; $\frac{162}{5}$ **OE**
	Total		**12**	
(b)(ii)	For **M1**, allow subtraction of "their" trapezium area from their |**(b)(i)** value| . Candidates may use $\int_{-1}^{2}(8x+14)\,dx=\left[4x^2+14x\right]_{-1}^{2}=16+28-4+14$ to earn **B1**. If $\int_{-1}^{2}(ax+b)\,dx$ is used for any line $y=ax+b$ to find the area of trapezium, then candidates are normally eligible for **M1** Candidates must find the area of a trapezium (and not a triangle) to earn **M1**			

Q4	Solution	Mark	Total	Comment
(a)	$(x+1)^2+(y-3)^2...$	M1 A1		one of these terms correct LHS correct with perhaps extra constant terms
	$(x+1)^2+(y-3)^2=50$	A1	3	
(b)(i)	$C(-1, 3)$	B1$\checkmark$	1	correct or **FT** from their equation in **(a)**
(ii)	$(r=)\sqrt{50}$	M1		correct or **FT** their $\sqrt{RHS}$ provided $RHS > 0$
	$=5\sqrt{2}$	A1	2	
(c)	$4^2+k^2+2\times4-6k-40=0$ **or** *"their"* $(4+1)^2+(k-3)^2=50$	M1		sub $x = 4$, correctly into given circle equation (or their circle equation)
	$k^2-6k-16(=0)$ **or** $(k-3)^2=25$	A1		
	$k=-2, k=8$	A1	3	
(d)	$D^2+1^2=$ " their r^2 "	M1		Pythagoras used correctly with 1 and r
	$D^2 = 50 - 1 = 49$ (distance =) 7	A1	2	Do not accept $\sqrt{49}$ or ±7
	Total		**11**	

(a) $(x--1)^2+(y-3)^2=\left(\sqrt{50}\right)^2$ scores full marks.

If final equation is correct then award 3 marks, treating earlier lines with extra terms etc as rough working.
If final equation has sign errors then check to see if **M1** is earned.

Example $(x+1)^2+(y-3)^2 - 40 + 1 + 9 = 0$ earns **M1 A1** but if this is part of preliminary working and final equation is offered as $(x+1)^2+(y-3)^2=50$ then award **M1 A1 A1.**

Example $(x-1)^2+(y-3)^2 = 50$ earns **M1 A0 ; Example** $(x-1)^2+(y+3)^2 = 50$ earns **M0**

(b)(ii) Candidates may still earn **A1** here provided RHS of circle equation is 50.

Example $(x-1)^2+(y+3)^2 = 50$ earns **M0** in **(a)** but can then earn **M1 A1** for radius = $\sqrt{50}=5\sqrt{2}$

If no $\sqrt{50}$ seen; " **(**radius =)$5\sqrt{2}$ **"** scores **SC2** .

(d) **NMS** (distance=) 7 scores **SC1** since no evidence that exact value of radius has been used.

A diagram with $\sqrt{50}$ or $5\sqrt{2}$ as hypotenuse and another side = 1 with answer = 7 scores **SC2**

Q5	Solution	Mark	Total	Comment
(a)	$\left(x+\frac{3}{2}\right)^2$	**M1**		$(x+1.5)^2$ **OE**
	$\left(x+\frac{3}{2}\right)^2-\frac{1}{4}$	**A1**	2	$(x+1.5)^2-0.25$ **OE**
(b) (i)	Vertex (–1.5, *) $(**, -0.25)$	**B1**√ **B1**√	2	strict **FT** "their" $-p$ strict **FT** "their" q Correct vertex is $(-1.5, -0.25)$
(ii)	$x=-1.5$	**B1**	1	**correct** equation in any form
(c)	$(x-2)^2+3(x-2)$ **or** $(x-2+\text{"their"}\ p)^2$	**M1**		replacing each x by x–2
	$y=(x-2)^2+3(x-2)+2 \quad +4$ **or** $y=(x-0.5)^2-0.25 \quad +4$ **OE**	**A1**		any correct unsimplified form with $y=\ldots+4$ or $y-4=\ldots$
	$y=x^2-x+4$	**A1cso**	3	
	Total		**8**	
(b)(i)	Accept coordinates written as $x=-1.5$, $y=-0.25$ **OE**			

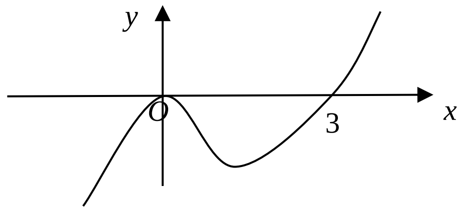
Q6	Solution	Mark	Total	Comment
(a)(i)	$(SA=)\ \ \pi r^2+2\pi rh$	B1		correct surface area
	$\pi r^2+2\pi rh=48\pi$ $\Rightarrow 2rh=48-r^2 \quad \Rightarrow h=...$	M1		equating *"their" SA* to 48π **and** attempt at $h=$
	$h=\dfrac{48-r^2}{2r}$	A1	3	**or** $h=\dfrac{24}{r}-\dfrac{r}{2}$ **OE**
(ii)	$V=\pi r^2 h=...$ $=\pi f(r)$	M1		correct volume expression & elimination of h using "their" **(a)(i)**
	$V=\pi r^2\left(\dfrac{48-r^2}{2r}\right)=24\pi r-\dfrac{\pi}{2}r^3$	A1	2	**AG (** *be convinced)*
(b)(i)	$\left(\dfrac{\mathrm{d}V}{\mathrm{d}r}=\right)\ 24\pi-\dfrac{3}{2}\pi r^2$	M1 A1	2	one term correct all correct, must simplify r^0
(ii)	$24\pi-\dfrac{3}{2}\pi r^2=0\Rightarrow r^2=\dfrac{48\pi}{3\pi}$	M1		"their" $\dfrac{\mathrm{d}V}{\mathrm{d}r}=0$ and attempt at $r^n=\ldots$
	$r=4$	A1		from correct $\dfrac{\mathrm{d}V}{\mathrm{d}r}$
	$\dfrac{\mathrm{d}^2V}{\mathrm{d}r^2}=-\dfrac{6\pi r}{2}$	B1√		FT "their" $\dfrac{\mathrm{d}V}{\mathrm{d}r}$
	$\dfrac{\mathrm{d}^2V}{\mathrm{d}r^2}<0$ when $r=4 \ \Rightarrow$ Maximum	A1cso	4	explained convincingly, all working and notation correct
	Total		**11**	

(a)(i) For **M1**, surface area must have two terms with at most one error in one of the terms.
Eg $\pi r^2+\pi rh=48\pi \ \Rightarrow h=...$ earns **M1**
It is not necessary to cancel π for **A1**

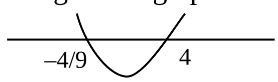
(a)(ii) May start again, eg using $2\pi rh=48\pi-\pi r^2 \ \Rightarrow 2\pi r^2h=48\pi r-\pi r^3 \quad \Rightarrow V=...$ etc for **M1**

(b)(ii) Award **B1√** for $\dfrac{\mathrm{d}^2V}{\mathrm{d}r^2}$ FT "their" $\dfrac{\mathrm{d}V}{\mathrm{d}r}$ only if $\dfrac{\mathrm{d}V}{\mathrm{d}r}=a+br^2,\ a\neq 0,\ b\neq 0$
For **A1cso** candidate must use all notation correctly, have correct derivatives and reason correctly.
Condone use of $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ etc instead of $\dfrac{\mathrm{d}^2V}{\mathrm{d}r^2}$ for **B1√** but not for **A1cso.**

May reason correctly using 2 values of r on either side of "their" $r=4$ substituted into V or $\dfrac{\mathrm{d}V}{\mathrm{d}r}$ for **B1√**
and if reasoning, working and notation are correct they may earn **A1 cso**.

Q7	Solution	Mark	Total	Comment
(a)		**M1**		cubic curve touching at O – one max, one min (may have minimum at O)
		A1		shape roughly as shown crossing positive x-axis
		A1	**3**	3 marked and correct curvature for $x < 0$ and $x > 3$
(b)(i)	$\text{p}(4) = 4^2(4-3)+20$	**M1**		p(4) attempted **or** full long division as far as remainder term
	(Remainder) $= 36$	**A1**	**2**	
(ii)	$\text{p}(-2) = (-2)^2(-2-3)+20$	**M1**		p(–2) attempted NOT long division
	$= 4\times(-5)+20 = 0$ *or* $-20+20=0$			working showing that p(–2) = 0
	therefore (x + 2) is a factor	**A1**	**2**	and statement
(iii)	x^2+bx+c *with* $b = -5$ **or** $c = 10$	**M1**		by inspection
	$(x+2)(x^2-5x+10)$	**A1**	**2**	must see product
(iv)	Discriminant of “their” quadratic $=(-5)^2-4\times10$	**M1**		be careful that cubic coefficients are not being used
	$-15<0$ so quadratic has no real roots	**A1cso**		
	(only real root is) –2	**B1**	**3**	independent of previous marks
	Total		**12**	

(a)	Award **M1** for clear ***intention*** to touch at O Second **A1:** allow curve becoming straight but withhold if wrong curvature in 1st or 3rd quadrants.
(b) (i)	May expand cubic as x^3-3x^2+20 Do not apply ISW for eg “ p(4) = 36, therefore remainder is – 36”
(ii)	Minimum required for statement is “so factor” Powers of –2 must be evaluated: **Example “**p(–2) = –8–12+20 = 0 therefore factor” scores **M1 A1** Statement may appear first : **Example “** x+2 is factor if p(–2) = 0 & p(–2) = –8–12+20 = 0” scores **M1 A1** However, **Example “** $\text{p}(-2) = (-2)^2(-2-3)+20 = 0$ therefore x+2 is a factor” scores **M1 A0**
(iii)	**M1** may also be earned for a full long division attempt, or a clear attempt to find a value for both b and c (even though incorrect) by comparing coefficients .
(iv)	Accept “ $b^2-4ac = 25-40<0$ so no real roots” for **M1 A1cso** Discriminant may appear within the quadratic equation formula “ $\sqrt{25-40}$ ” for **M1**

Q8	Solution	Mark	Total	Comment
(a)	$x^2+(3k-4)x+13=2x+k$ $x^2+3kx-6x+13-k=0$ $x^2+3(k-2)x+13-k=0$	**B1**	**1**	at least one step such as this line **AG (**be convinced)
(b) (i)	$\{3(k-2)\}^2-4(13-k)$	**M1**		correct discriminant
	$9(k^2-4k+4)-52+4k$	**A1**		correct and brackets expanded correctly
	<0 $9k^2-32k-16<0$	**A1cso**	**3**	condition must appear before final answer **AG** Penalise poor use of brackets here even if candidate recovers
(ii)	$(9k+4)(k-4)$	**M1**		correct factors or correct use of formula as far as $\dfrac{32\pm\sqrt{1600}}{18}$
	CVs are $-\dfrac{4}{9}$ and 4	**A1**		condone equivalent fractions here
	+ – + $-\dfrac{4}{9}$ 4	**M1**		use of sign diagram or graph 
	$-\dfrac{4}{9}<k<4$	**A1**	**4**	fractions must be simplified for final mark
	Total		**8**	
	TOTAL		**75**	

(b)(i) For **M1** must be attempting to use b^2-4ac but ***condone poor use of brackets***.

(b)(ii) For second **M1**, if critical values are correct then sign diagram or sketch must be correct ***with correct CVs marked***.
However, if CVs are not correct then second **M1** can be earned for attempt at sketch or sign diagram but ***their CVs*** MUST be marked on the diagram or sketch.
Final **A1,** inequality must have *k* and no other letter.

Final answer of $k<4$ AND $k>-\dfrac{4}{9}$ (with or without working) scores 4 marks .

(A) $-\dfrac{4}{9}<x<4$ (B) $k<4$ OR $k>-\dfrac{4}{9}$ **(**C) $k<4$, $k>-\dfrac{4}{9}$ **(D)** $-\dfrac{4}{9}\le k\le 4$
with or without working each score 3 marks (**SC3**)

Example NMS $\dfrac{4}{9}<k<4$ scores **M0 (**since one CV is incorrect)

Example NMS $k<\dfrac{72}{18}$, $k<-\dfrac{8}{18}$ scores **M1 A1 M0 (**since both CVs are correct)